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# ENERGY-CONTAINING-RANGE MODELING OF FULLY-DEVELOPED CHANNEL FLOW USING A HYBRID RANS/LES TECHNIQUE

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## Abstract

Turbulent channel flow is used to study energy-containing-range modeling using a hybrid RANS/LES approach. The hybrid model relates the mean component of an LES-type subgrid diffusivity to the turbulence diffusivity from RANS via a transfer function. Details of this transfer function in the energy-containing-range of the turbulence are shown to be very important when modeling coarsely resolved flows. Three transfer function models are compared. One interpolates the turbulence diffusivity between the LES and RANS limits using an algebraic blending. The second uses von Karman's empirical fit to the turbulent kinetic energy spectrum to diagnose energy-containing-range structure. The third uses a modified Smagorinsky subgrid model corrected to have the proper mean time scale as diagnosed from RANS. Our Reynolds number is 640, based on the channel half height and on the friction velocity. Comparison of mean-field and root-mean-square statistics to other studies clearly identify the mean time-scale model as the best performer.

## 1. Introduction

Reynolds-Averaged Navier-Stokes (RANS) modeling is the contemporary workhorse for CFD. The demands on a RANS model can be severe, since it must accommodate the geometric scales of a flow under complicated conditions. Most RANS models perform well for flows similar to the ones for which they are tuned but fail, often severely, for other flows. Large-eddy simulation (LES), as an alternate technology, resolves the geometric scale of a flow modeling only the effects of the inertial range scales. The cost of a LES can be quite high because of high grid density and the need to collect statistics from instantaneous solutions.

Recent work proposes a middle ground between RANS and LES. Spalart, et al. (1997) and Speziale (1998) have shown that a RANS code can be used to give LES-like solutions if the RANS eddy diffusivity is decremented appropriately using a grid-resolution-dependent transfer function. It is constructed so that in the coarse-grid limit, when no turbulence fluctuations are resolved, the model becomes RANS. Similarly when the energy-containing-range scales are resolved, the model becomes a Smagorinsky-type LES. Peltier, Zajaczkowski, and Wyngaard (2000) implemented a hybrid RANS/LES model based on that idea.

We evaluate three candidate transfer functions and select from them a “best” choice. Fully-developed channel flow at Reynolds number 640, based on channel half height and on friction velocity, is used as the test case, since the expected results are well known from the literature.

## 2. Mathematical formulation

### 2.1 The Transport Budgets

The filtered, incompressible Navier-Stokes equations are solved for the resolvable scales of fully-developed turbulent channel flow. The flow is divergence free to enforce continuity. The equations are

$$\tilde{u}_{i,i}^r + (\tilde{u}_i^r \tilde{u}_j^r)_{,j} = -\tilde{p}_{,i}^r + \frac{1}{\text{Re}_\tau} \tilde{u}_{i,jj}^r - \tilde{\tau}_{ij,j}^{SGS} - 1 \quad \text{and} \quad \tilde{u}_{i,i}^r = 0. \quad (1)$$

The superscript “r” refers to “resolvable scale”. The capping tilde is used to denote a variable with both mean and fluctuating parts. The “-1” on the right side is the mean pressure gradient nondimensionalized on the channel half-height and on the friction velocity. The pressure gradient term on the right side of (1) is the deviation from the mean gradient.  $\text{Re}_\tau$  is the appropriate Reynolds number. Noslip conditions are enforced at the lower and upper walls of the channel. Wall functions are not used. The streamwise and cross-stream directions are periodic.

### 2.2 The Subgrid Model

We use an eddy diffusion model for the subgrid stress in (1):  $\tilde{\tau}_{ij} = -2\bar{v}_T^{SGS} \tilde{S}_{ij}^r$ , where  $\tilde{S}_{ij}^r = (\tilde{u}_{i,j}^r + \tilde{u}_{j,i}^r)/2$  is the resolvable-scale strain-rate tensor. Contributions to the eddy diffusivity are from direct interactions with the mean flow and interactions within the fluctuating field,  $\bar{v}_T^{SGS} = \bar{v}_T^{MS} + \bar{v}_T^{FL}$ . Peltier and Zajaczkowski (Reg. Paper #81 of this conference) show that  $\bar{v}_T^{FL} \gg \bar{v}_T^{MS}$  in the fine-grid limit (LES). Traditional LES subgrid models (like the Smagorinsky model) already perform well for filter scales in the inertial range, so no additional work must be done to identify a suitable model for  $\bar{v}_T^{FL}$ . In the coarse grid limit,  $\bar{v}_T^{FL} \ll \bar{v}_T^{MS}$  (Peltier and Zajaczkowski, Reg. Paper # 81), so direct interactions with the mean straining field are of primary importance.  $\bar{v}_T^{MS}$  can be inferred from RANS:

$$\bar{v}_T^{MS} = T(\ell, \Delta, \eta) \bar{v}_T^{RANS}. \quad (2)$$

$T(\ell, \Delta, \eta)$  is an appropriate grid-resolution-dependent transfer.

A diffusivity is the product of the characteristic length and velocity scales for the subgrid turbulence. For RANS,  $\bar{v}_T^{RANS} = q \ell$ , where  $\ell$  is the dissipation length and

$q$  is the square root of the turbulence kinetic energy. Note: the familiar coefficient,  $C_\mu = 0.09$ , and any viscous damping function are absorbed in our definition of  $\ell$ . Similarly, for a partially resolved flow field with characteristic length and velocity scales,  $\Delta$  and  $v$ , the mean diffusivity is  $\bar{v}_T^{MS} = v \Delta$  giving

$$T(\ell, \Delta, \eta) = \frac{v \Delta}{q \ell} \quad (3)$$

for the transfer function. Again, coefficients like the Smagorinsky constant,  $C_s = 0.065$ , and damping functions are absorbed in  $\Delta$ , a filter scale proportional to the characteristic length of a local grid-cell volume. The velocity scale,  $v$ , is the square root of the subgrid turbulence kinetic energy. Dependence of (3) on the Kolmogorov scale,  $\eta$ , arises because of the viscous cutoff scale for turbulence near  $\eta$ .

We use the traditional Smagorinsky eddy-diffusivity closure of LES whose velocity scale is inferred from the strain-rate invariant,  $v = \Delta \cdot S$ . The Smagorinsky model breaks down under coarse resolution, apparently because this velocity scale is inappropriate in the energy containing range. Our modeling effort seeks to correct the Smagorinsky closure by defining a more appropriate energy-containing-range velocity scale:

$$\tilde{v}_T^{SGS} = \tilde{v}_T^{SMAG} + T(\ell, \Delta, \eta) (\bar{v}_T^{RANS} - \bar{v}_T^S) \quad (4)$$

where  $\bar{v}_T^{MS} \equiv \Delta (\Delta \bar{S})$ , the Smagorinsky model applied to the mean flow.  $\bar{S} = (U_{i,j} + U_{j,i})/2$  is the mean-field strain-rate invariant (from RANS). Three models for  $T(\ell, \Delta, \eta)$  are evaluated in this study.

### Model 1

Following Peltier et al. (2000), we diagnose  $v$  by integrating the inertial-range form for the turbulence kinetic energy spectrum,  $E(\kappa) = \frac{8}{3} \varepsilon^{2/3} \kappa^{-5/3}$  between the filter-scale wavenumber,  $\kappa_\Delta = 2\pi/\Delta$ , and a dissipation-range cutoff wavenumber,  $\kappa_\eta = 2\pi/\Delta_\eta$ , where  $\Delta_\eta \approx 0.1\eta$  (see Hinze, 1975, p. 224). Denoting the inertial-range form of the transfer function  $I(\ell, \Delta, \eta)$ , we blend between the inertial range and RANS via:

$$T(\ell, \Delta, \eta) = \left[ \frac{I(\ell, \Delta, \eta)^2}{1 + I(\ell, \Delta, \eta)^2} \right]^{1/2}. \quad (5)$$

The power of 2 used in the blending function comes from Peltier et al.'s (2000) preliminary optimization of the model.

*Model 2*

By integrating von Karman's empirical fit to the turbulence kinetic energy spectrum from  $\kappa_\Delta$  to  $\kappa_\eta$ , we can compute  $T(\ell, \Delta, \eta)$  directly. Von Karman's spectrum is given by (Hinze, 1975, p. 244):

$$E(\kappa) = C_{vK} \frac{(\kappa/\kappa_e)^4}{[1 + (\kappa/\kappa_e)^2]^{17/6}}. \quad (6)$$

The coefficient  $C_{vK}$  and the wavenumber  $\kappa_e$  are set by requiring (6) to have the proper inertial range amplitude,  $\frac{8}{3}\varepsilon^{2/3}$ , in the limit of very large wavenumbers and by requiring the spectrum to integrate to the turbulent kinetic energy,  $q^2$ ; the dissipation rate,  $\varepsilon$ , and the turbulent velocity scale,  $q$ , are provided by RANS. One drawback to using (6) is that an inertial range will be imposed for all Reynolds numbers, even very low Reynolds numbers for which an inertial range does not exist.

*Model 3*

Our final model adopts the Smagorinsky length scale for  $\Delta$  but replaces the incorrect time scale in (4),  $\bar{S}^{-1}$ , with the turbulence time scale from RANS,  $\ell/q$ :

$$\bar{v}_T^{MS} \approx \Delta \left( \Delta \frac{q}{\ell} \right) \rightarrow T(\ell, \Delta, \eta) = \left( \frac{\Delta}{\ell} \right)^2. \quad (7)$$

**3. Numerical Method**

A finite difference discretization of Eq. (1) with discretized boundary conditions and turbulence modeling is solved. The solution procedure follows the fractional step approach outlined by Rai and Moin (1991); however, a linear blending of second-order accurate weighted-average central differencing with first-order accurate upwind differencing is used for the nonlinear advection terms for values of the transfer function greater than 0.9. This range was chosen by numerical experiment emphasizing the need to support turbulence scales of motion while retaining stability for very coarse grids. Explicit dependence of the blending on cell Peclet number was not used. The code was validated based on the previous study by Peltier et al. (2000) and based on comparisons to other experimental and numerical data.

**4. Numerical Results**

We use fully developed channel flow at  $Re=640$  based on the friction velocity and on the channel half height as our test problem because this case is common in the literature and is a simple flow field that emphasizes all of the difficulties inherent in modeling wall bounded flows. The domain size is  $2\pi \times \pi \times 2$ , similar to cases

studied by Moin & Kim (1982), and our grid resolution is  $42 \times 23 \times 65$ . The wall-normal direction is aligned with our  $z$  coordinate, which uses hyperbolic tangent stretching toward the solid boundaries. The near-wall spacing is prescribed to give  $y^+ = 1$  for the second grid point. Four cases are compared: our three transfer functions and a baseline case using a traditional Smagorinsky subgrid model. Since the RANS statistics are stationary, the RANS input data is computed *apriori*.

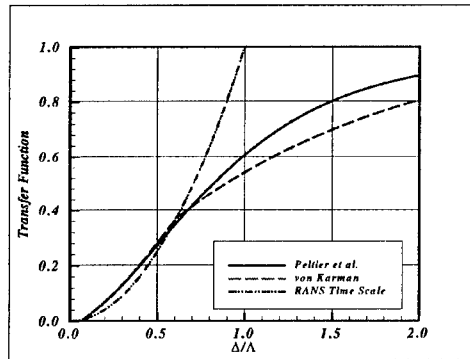


Figure 1  $T(\ell, \Delta, \eta)$  vs  $\Delta/\ell$

Figure 1 presents the three candidates for  $T(\ell, \Delta, \eta)$  plotted as a function of  $\Delta/\ell$ . The Peltier et al. (2000) and von Karman formulations transition to RANS much slower than the mean time scale model (Model 3). For grid resolutions giving  $\Delta/\ell = 1$ , the mean time-scale model becomes RANS, whereas, the Peltier et al. (2000) and von Karman models use only 40% and 60% of the RANS diffusivity.

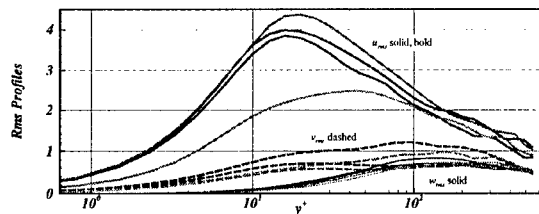


Figure 3 Profiles of root-mean-square velocity

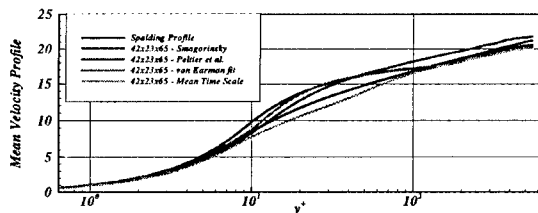


Figure 2 Mean velocity profiles plotted against

Mean-field statistics are presented in Figs. 2 and 3. The Spalding profile is included in Fig. 2 for reference. The Smagorinsky baseline case overshoots the Spalding profile in the buffer layer but recovers a log-law slope in the logarithmic layer. The Peltier et al. (2000) and von Karman models also overshoot the Spalding Profile in the buffer layer, then recover toward the channel core. The mean time scale model tracks the Spalding Profile well across the domain though a mild undershoot is apparent in the buffer layer.

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The peak values of our root-mean-square (rms) statistics (Fig. 3) agree well with observations from Moin & Kim (1982): 2.4, 1.3, and 0.9 for the longitudinal, cross-stream, and wall-normal rms velocities. Moin & Kim (1982) show that the horizontal velocity rms values peaks near  $y^+ \approx 30$  in agreement with our mean time-scale model. The peak location for the transfer functions based on Peltier et al. (2000) and the von Karman spectrum occurs near  $y^+ \approx 20$ . Moin & Kim show that the cross-stream and wall-normal velocity rms peak locations occur near  $y^+ \approx 120$ , similar to our results. Only the mean time-scale model yields an rms value for the horizontal velocity at the peak near the 2.4 presented by Moin & Kim (1982). Each of the other models at our grid resolution overshoot the target value by between 58% to 83%. It is difficult to distinguish significant quantitative differences for the other rms components.

## 5. Conclusions

Three transfer functions relating the mean turbulence diffusion from direct interactions with the mean straining field to the eddy diffusivity from RANS were evaluated. They represent 1) a simple blending to interpolate between the RANS and LES limits, 2) use of von Karman's spectrum to diagnosis the proper velocity scale, and 3) a correction of the mean time scale from the Smagorinsky model using the turbulence time scale from RANS. Our results showed clearly that *Model 3* agreed best with accepted physics. We recommend that *Model 3* be used for future work using this hybrid RANS/LES approach.

## Acknowledgments

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